Valuing Debt and Leases with Optimal Capital Structure

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Abstract

This paper shows how to value debt or a debt equivalent, particularly emphasizing leases, when the borrowing firm has an optimal capital structure. In contrast to the standard results of the leasing literature, the lease valuation methodology depends upon the determinants of the optimal capital structure. Specifically, when capital structure is determined by the tradeoff between the corporate tax benefit and agency costs of debt, the standard approach of using the after-tax borrowing rate to discount a lease is incorrect. Instead, it is appropriate to discount using the pretax borrowing rate, with an additional term to reflect marginal agency costs. However, when capital structure is determined by the tradeoff between the corporate tax benefit of debt and the personal tax benefit of equity, the standard approach is correct. The calculations required for each valuation methodology (corresponding to a different determinant of the optimal capital structure) are straightforward to implement.

1. Introduction

The literature on leasing (Myers, Dill, and Bautista (1976) and Franks and Hodges (1978)) has traditionally analyzed the value of a financial lease to the leasing firm (lessee) by employing two rules. The first is a principle: a lease contract is essentially a form of debt (a debt equivalent). Since using a lease is a substitute for using other forms of debt available to the firm, the relative value of the lease to the leasing firm must recognize the alternative of using ordinary debt[1] that the lease displaces. The second is really an assumption: debt can be employed at the margin with full tax shielding benefits intact. Immediately derivable from this assumption is that the time value of money implied by ordinary debt, applicable to after-tax debt-related cash flows, is given by the after-tax borrowing rate. Together, these two rules imply that the relative [present] value of the rental payments associated with the lease is found by discounting the after-tax lease payments by the after-tax borrowing rate.

However, the second rule of this valuation approach is potentially at odds with the tradeoff theory of capital structure, in which, at the optimal capital structure, the corporate tax shielding benefits of debt financing may be reduced (DeAngelo and Masulis (1980)), fully offset by personal tax effects (Miller (1977)), or offset by financial distress effects such as agency costs (Jensen and Meckling (1976), Myers (1977)) or bankruptcy costs. If debt generates an additional impact upon corporate cash flow beyond corporate tax shielding, or if the tax shields are not fully effective, then the standard valuation result of the leasing literature need not apply. More specifically, if the assumption of the standard valuation theory for leases (full tax shielding benefits) is inconsistent with an assumption of optimal corporate capital structure, the appropriateness of the standard leasing valuation results are called into question.

This paper analyzes the relative value of a financial lease in the context of a firm employing an optimal capital structure. Although the precise formulation of the derived results on leasing valuation varies, depending on which factors drive the optimal tradeoff between debt and equity financing, each of the derived results is highly tractable. Four formulations of optimal capital structure are considered in this paper, and the appropriate

[1] Ordinary debt refers to debt with interest payments charged at the prevailing market interest rate.

leasing valuation results are derived in each case. First, a firm with full tax shielding is examined. Second, a firm trading off tax benefits with agency costs of debt is considered. Third, the case of a firm trading off uncertain corporate tax benefits and certain personal tax costs (as in DeAngelo and Masulis) is examined. Fourth, the case of the Miller equilibrium is looked at, wherein the firm has no net tax benefit of debt. In the first case, it is shown that leases should be valued by discounting pretax cash flows at the pretax borrowing rate. In the second, proper valuation involves discounting after-tax cash flows at the pretax borrowing rate, with an additional term, a downward adjustment to account for agency impact over the life of the lease. This calculation is straightforward and easily to implement. In the third case, after-tax cash flows should be discounted at the after-tax borrowing rate, with the important caveat that the effective marginal tax rate (which generally differs from the statutory tax rate) must be used. (Methods for computing these effective rates are developed and described in Shevlin (1990), Graham (1996a, 1996b) and Graham, Lemmon and Schallheim (1998)). Only in the fourth case, the Miller equilibrium, does the standard approach from the literature completely apply, and after-tax cash flows may be discounted at the after-tax borrowing rate.

Valuing a financial lease is a particular example of a more general problem, that of valuing a debt (e.g., subsidized debt carrying a below-market interest rate) or a debt equivalent (e.g., a leasing arrangement) contract, characterized by a schedule of future principal and/or interest payments. For the case of a lease, because of their tax-deductibility, lease payments are treated as equivalent to a series of interest payments.

Because of the tax shielding benefit of a debt or debt equivalent, there are two different valuations associated with debt: the market value (from the viewpoint of the lender, or debtholder), and the relative, or net, value (from the viewpoint of the borrower, corporation or equityholder). The former is simply the familiar calculation of discounting the pretax debt payments, whether interest or principal, at the fair borrowing rate. However, the latter calculation requires more care. In light of the first rule above, the net value to the firm must recognize that ordinary debt is being displaced, and therefore requires calculating the present value of the debt or debt equivalent relative to the alternative of the displaced ordinary debt [and its associated tax savings]. Indeed, discounting the after-tax debt payments of any ordinary debt at the after-tax borrowing rate (implicitly assuming the second rule above) gives zero, regardless of its repayment schedule.[2]

However, if the cash flows (from the borrower's viewpoint) associated with a debt or debt equivalent are not completely characterized by interest and principal payments to the lender and fully effective associated tax shielding (for example, if there are agency costs associated with debt, reducing the expected cash flows from operations), then interest and principal payments and fully effective tax shields are not the relevant cash flows for the firm to discount, and a different approach is needed.

We emphasize that the results of this paper can depart from the standard valuation results in the leasing literature due to our assumption of an optimal corporate capital structure. The leasing literature assumes fully effective tax shielding at the margin, with no negative impact of debt on corporate cash flow. The key observation this paper makes is that, at an optimal capital structure, these assumptions are not typically consistent with an optimal capital structure as normally envisaged (the sole exception is the Miller equilibrium). Tax shielding may be less than fully effective, tax shielding may be exhausted, or financial distress effects may have a negative impact upon [future] operating cash flows. All of these lead to different valuation results than that found in the standard literature. Furthermore, in the important case that the capital structure is determined by a tradeoff between agency costs and tax benefits of debt, the appropriate calculation requires using the pretax rather than after-tax borrowing rate.

Each of the remaining sections of this paper considers a different formulation of optimal capital structure and the resulting implication for valuing leases: the case of full shielding in Section 2, the case where financial distress costs trade off against the tax benefits of debt in Section 3, and finally the case where personal tax benefits of equity counterbalance corporate tax benefits of debt (including both the DeAngelo/Masulis and Miller equilibria) in Section 4. Each section examines the general problem of determining the relative valuation for a debt or debt equivalent, with valuing a lease

[2] For single-period debt at the fair borrowing rate, it is easy to directly show that discounting the after-tax cash flows (received principal, repaid principal, paid interest) at the after-tax borrowing rate gives a present value of zero. Since any longer-lived debt schedule can be decomposed into a series of single-period debts, a present value of zero also applies to ordinary debt with any repayment schedule.

illustrated as a special case. Section 5 examines lease valuation when borrowing is constrained by limited collateral of the firm.

2. The case of full shielding

The example in this section illustrates that, when operating at the optimal capital structure, financing risk-free cash flows with fully deductible debt may not even be possible; here, evaluating a lease by discounting after-tax lease payments at the after-tax borrowing rate is inappropriate.

Consider a firm with risk-free (certain) pretax operating cash flows C_t in period t, growing at a constant rate $g > 0$ each period.[3] Facing a corporate tax rate τ and risk-free interest rate r, the firm can fully shield itself from corporate tax with a capital structure utilizing debt $D_t^* = C_{t+1}/r$ in period t. As shown in Berens and Cuny (1995), although the firm is less than 100% debt financed, it is fully shielded from taxes at all times. This capital structure is optimal, since taxes are completely avoided. The firm value in period t is $V_t^* = C_{t+1}/(r - g)$.

Now suppose the firm has a debt equivalent in place, entailing future commitments of principal payments P_s, tax-deductible interest payments I_s, and additional (non-interest) tax deductions δ_s at future dates indexed by s. (In particular, this could be a leasing arrangement. Since lease payments are tax deductible, I_s can then be interpreted as the agreed lease payments, δ_s as depreciation foregone by leasing instead of purchasing, while P_s is zero.) Since the firm can only utilize marginal tax deductions when taxable income is positive, ordinary debt at time t is displaced by next-period interest commitments I_{t+1} and other tax deductions δ_{t+1}. The amount of debt (in conjunction with the debt equivalent) now required to fully shield corporate cash flows from taxes is $D_t^{**} = (C_{t+1} - I_{t+1} - \delta_{t+1})/r \leq D_t^*$, which implies $rD_t^{**} + I_{t+1} + \delta_{t+1} = C_{t+1}$ for all t; interest on debt plus interest and other tax deductions from the debt equivalent fully

[3] Although the constant growth rate is unimportant for our debt valuation results (all that is required is positive growth, guaranteeing full tax shielding with less than 100% debt financing), the notation is then kept consistent with Berens and Cuny (1995).

shield operating cash flows.[4] Since the firm is fully shielded from taxes, the value of the firm (excepting the debt equivalent claim) at period t is

$$V_t^{**} = \sum_{s>t} (C_s - P_s - I_s)/(1 + r)^{s-t} = V_t^* - \sum_{s>t} (P_s + I_s)/(1 + r)^{s-t}.$$

Thus, the net value of a debt equivalent (from the corporate or stockholder viewpoint) is found by discounting pretax interest and principal payments at the pretax borrowing rate. In the case of a lease, pretax lease payments are discounted at the pretax borrowing rate. Here, the optimal debt level implies that operating cash flows are fully shielded. In particular, there is no remaining debt capacity, in the sense that any additional marginal debt would generate no shielding benefit. Thus, the assumption that marginal risk-free cash flows can be fully financed by *debt which carries full shielding benefits* breaks down here. Valuing a lease or other debt equivalent by discounting after-tax lease payments at the after-tax borrowing rate is inappropriate.

3. The case of agency costs

We start with a simple example to illustrate why the standard argument in the literature, using ordinary debt to replicate a lease, fails when the optimal capital structure is determined by a tradeoff of corporate tax benefits and agency costs of debt. Suppose the discount rate is 10% and the corporate tax rate is 40%. Consider a one-period lease with lease payment \$10. This generates a single tax savings, \$4 next period. The present value of this lease payment is \$9.09.

The only loan with exactly the same tax benefit is a one-period loan with a \$10 interest payment and \$100 principal. Of course, the present value of next period's \$110 loan payment is \$100. This loan and the lease have exactly the same tax benefits and lifetime, but the magnitude of the debtholder claim with the loan is over ten times that with the lease, and can therefore be expected to generate much higher agency costs.[5] Thus, although the benefits of the lease and loan can be matched, their costs will not. (By

[4] This assumes that the debt equivalent is not so large in magnitude that an interior optimal capital structure is no longer obtained.

a similar argument, aligning the costs of the loan and lease would leave the benefits mismatched.)

Fortunately, when capital structure is an interior optimum, we do not need to rely on an argument based on perfect substitutability between leases and loans. At the margin, the benefits and costs of an ordinary loan are equated (providing zero net value to the firm), and the relative value of a lease (based on its benefits and costs) can therefore be inferred.

Let V_t be the market value of the firm (ex-current period cash flow), D_t the market value of the debt, and C_t be the pretax expected operating cash flow of the firm in period t. Agency costs are modeled by allowing the upcoming operating cash flow C_{t+1} to depend upon the market value of total debt: the level of ordinary debt D_t plus any debt equivalent extant at time t. It is assumed that agency issues have an adverse impact upon operating cash flows, at an increasing rate ($C' \leq 0$, $C'' \leq 0$).[6] For simplicity, investor risk neutrality is assumed. With risk-free rate r and corporate tax rate τ, in the absence of any debt equivalents, the optimal capital structure solves

$$V_t^* = \underset{D_t \geq 0}{\text{Max}} \quad ((1 - \tau)C_{t+1}[D_t] + \tau r D_t + V_{t+1}^*)/(1 + r). \qquad (1)$$

The optimal market value of the firm in current period t, V_t^*, is thus the discounted value of: next period's after-tax operating cash flow C_{t+1} (dependent upon period t debt), plus the tax savings $\tau r D_t$ associated with next period's interest payment rD_t, plus the continuation value of the firm.[7] Denoting the optimal debt level by D_t^*, the first-order condition is $(1 - \tau)C_{t+1}'[D_t^*] + \tau r = 0$.

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[5] As explained below, the relative advantage of the lease arises because lease payments are fully tax-deductible, whereas only interest payments, and not principal payments, on a loan are.

[6] In agency cost stories, the presence of debt causes a wedge between the equity claim and total firm claim. This may lead to distortions, including the acceptance of negative NPV projects, the rejection of positive NPV projects, and preference of lower NPV projects with shorter lives. Thus, agency costs imply that debt has a negative impact on future cash flows.

[7] Using rD_t as the interest payment assumes the debt is riskless. Strictly speaking, debt cannot generate an agency distortion unless it is risky. It is shown in the Appendix that this section's results can also be derived with risky debt.

Now assume the presence of debt equivalent with principal payments P_s, tax-deductible interest payments I_s, and additional tax deductions δ_s at future dates $s > t$. At time t, the market value of future payments is

$$Q_t = \sum_{s>t} (P_s + I_s)/(1+r)^{s-t}, \tag{2}$$

the present value of these claims from the lender viewpoint. In the presence of this debt equivalent, optimal firm value (excepting the debt equivalent claim) is

$$V_t^{**} = \underset{D_t \geq 0}{\text{Max}} \Big((1-\tau)C_{t+1}[D_t + Q_t] + \tau(rD_t + I_{t+1} + \delta_{t+1}) - (P_{t+1} + I_{t+1}) + V_{t+1}^{**}\Big)/(1+r). \tag{3}$$

Denoting the optimal debt level by D_t^{**}, the first-order condition is given by $(1-\tau)C_{t+1}'[D_t^{**} + Q_t] + \tau r = 0$. The first-order conditions of (1) and (3) imply that $D_t^{**} + Q_t = D_t^{*}$, thus the presence of the debt equivalent displaces ordinary debt. The net value[8] from the firm's (or equityholders') viewpoint of this outstanding debt equivalent is therefore (see Appendix for details)

$$\begin{aligned} NV_t &= V_t^{**} - V_t^{*} \\ &= \sum_{s>t} \big(-P_s - (1-\tau)I_s + \tau\delta_s \big)/(1+r)^{s-t} \\ &\qquad - \tau r/(1+r) \cdot \sum_{s>t} (s-t)(P_s + I_s)/(1+r)^{s-t}. \end{aligned} \tag{4}$$

This net value recognizes benefits and costs associated with the debt equivalent, as well as the lost opportunity cost of the ordinary debt crowded out by the debt equivalent. This result is summarized in the following proposition.

Proposition 1. *Suppose a firm is at its interior optimal capital structure, determined by a tradeoff of corporate tax benefits and agency costs of debt. At the margin, the net value to the firm, including tax benefits and agency costs, of an outstanding debt [equivalent] contract can be found by discounting, using the (pre-tax) borrowing rate: future*

[8] As the debt equivalent is essentially a claimant on the firm, said value is typically negative.

principal payments, future after-tax interest payments, future tax savings from any additional deductions, and $\tau r/(1 + r)$ *times time-weighted future payments, as in equation (4).*

Proof: See Appendix.

Equation (4) characterizes the relative value to the equityholders of the presence of the debt equivalent, recognizing that any debt equivalent displaces ordinary debt. This naturally breaks into three parts. Obviously, the commitment to make future principal payments P_s and interest payments I_s is a cost to the equityholders. These are adjusted to reflect the future tax benefits of debt τI_s and any tax savings from other deductions $\tau \delta_s$. However, there is an additional cost due to agency effects. Although marginal agency costs are typically difficult to measure in practice, they are quantifiable at an interior optimum capital structure since they are equated with the quantifiable marginal benefits of ordinary debt. In each future period, agency costs equal $\tau r/(1 + r)$ times the market value of extant debt at that point.[9] Equivalently, the present value of all agency costs generated across time by a single future (principal or interest) payment equals the product of $\tau r/(1 + r)$, the length of time until the payment is made, and the present value of the payment. Thus, agency costs associated with each payment are proportional to the time until that payment is made. Since the firm is at an interior optimal capital structure, equating marginal benefits and costs of ordinary debt, consistency of this valuation methodology should imply that, as a special case, any issuance of ordinary debt at the market price (2) should be value neutral. This is confirmed in the following corollary.

Corollary A. *Entering into any debt contract with interest rate* r *at the market price (2) has zero net present value to the firm, regardless of repayment schedule.*

Proof: See Appendix.

Although borrowing at the market interest rate has zero net value to the firm, entering into a debt contract with an advantageous or disadvantageous (below or above-market) interest rate need not have zero net value, even though it displaces ordinary debt, as the following two examples show.

[9] In a continuous time version of this result, the factor $\tau r/(1 + r)$ reduces to τr.

For the first example, consider permanent debt with principal X and interest rate $r_d \neq r$. Employing equation (4) with $P_s = 0$, $\delta_s = 0$, and $I_s = r_d X$ for all $s > t$ implies the net value of such a contract in place is $V_t^{**} - V_t^* = -r_d X/r$. Therefore, borrowing X through such a debt contract creates net value $+X - r_d X/r = (r - r_d)X/r$ for the firm. This result happens to coincide with that achieved by discounting after-tax cash flows at the after-tax borrowing rate.

For the second example, consider one-period debt of principal X with interest rate r_d. Using equation (4) with $P_{t+1} = X$, $I_{t+1} = r_d X$, $P_s = I_s = 0$ for $s > t + 1$, and $\delta_s = 0$ implies that borrowing X through such a debt contract creates value $(1 + r - \tau)(r - r_d)X/(1 + r)^2$. For $r_d < r$, this is greater than the result achieved by discounting after-tax cash flows at the after-tax borrowing rate.

Depending upon the price, a leasing arrangement may potentially be advantageous to the firm. Since lease payments are tax deductible, while only interest (and not principal) payments on debt are tax deductible, leasing generally offers the firm more tax deductibility than by using ordinary debt with similar payments. Agreeing to lease at time t (rather than purchasing at price Π_t), with future lease payments I_s, and foregone depreciation δ_s (so that $I_s \geq 0$ and $\delta_s \leq 0$), generates net value to the firm of

$$NV_t = \Pi_t + \sum_{s>t} \left(-(1 - \tau)I_s + \tau\delta_s - (s - t)\tau r I_s/(1 + r) \right)/(1 + r)^{s - t} \qquad (5)$$

at time t. This result, which follows directly from (4) by setting principal payments P_s to zero, recognizes the effect of crowding out ordinary debt with the lease. This is summarized in the following corollary.

Corollary B. *Suppose a firm is at its interior optimal capital structure, determined by a tradeoff of corporate tax benefits and agency costs of debt. At the margin, the net value to the firm, including tax benefits and agency costs, of an outstanding leasing arrangement can be found by discounting, using the (pre-tax) borrowing rate: future after-tax lease payments, foregone future tax savings from depreciation, and* $\tau r/(1 + r)$ *times time-weighted future lease payments, as in equation (5).*

Proof: See Appendix.

Notice that this result stands in contrast to the results of Myers, Dill and Bautista and of Franks and Hodges. Their results assume that a firm realizes the full benefit of tax shields without negative impact upon cash flow. This assumption is inconsistent with tradeoff models of optimal capital structure driven by agency costs offsetting the tax benefit of debt.

As a particularly relevant example, consider the special case of a T-period leasing arrangement with periodic lease payments L, rather than purchasing the asset at price Π, and foregoing depreciation ($\delta_s \leq 0$). From equation (5), with $I_s = L$ for $s - t \leq T$, $I_s = 0$ for $s - t > T$, and $\delta_s = 0$, the net value to the firm of the future lease payments is

$$NV_t = \Pi + \tau \cdot PV(\delta_s\,;\, r) - L/r + (L/r)(1 + r)^{-T} + T\tau L(1 + r)^{-(T+1)}. \qquad (6)$$

The lease payment terms have the following intuition. Issuing T-period coupon debt with principal L/r and coupon L would create zero net present value for the firm, as it is ordinary debt. Alternatively, the net value to the firm of newly issued coupon debt equals -L/r, exactly offsetting the principal just received. Relative to this coupon debt, the lease has two effects. First, no principal L/r need be repaid at time T. Second, no agency costs are generated by outstanding principal in the interim T periods. These are all reflected in the respective terms of (6).

The net value of the T-period lease depends upon the length of the lease. With coupon debt, the interest but not the principal payments generate tax deductions for the firm. However, with the lease, all lease payments generate tax deductions. For a relatively short-lived lease, this can offer a major relative tax advantage: the lease resembles coupon debt whose principal (thus the bulk of its payments) has somehow become tax deductible. However, for a very long-lived lease, this offers little advantage: the lease resembles ordinary coupon debt, since the principal payment on the coupon debt is far off and has relatively small present value. Thus, as T becomes large, (6) converges to -L/r, the net value of the outstanding coupon debt.

This section assumed that agency issues impact firm value through reducing operating cash flow in the period the debt is in place, and took debt to be riskless. In the Appendix, both of these assumptions are relaxed. We allow the agency or other financial distress cost impact to include a reduction in corporate cash flows in the future, and we allow debt to be risky. Neither of these extensions changes the valuation results of this section in a meaningful way.

4. The case of personal taxes

This section analyzes the case where the optimal capital structure is driven by the tradeoff between the personal tax disadvantage and the corporate tax advantage of debt. A critical way in which this differs from the model of Section 3 is that debt generates no adverse effect on corporate cash flow. Rather, the cost of debt to the firm is manifested in a higher borrowing rate (generated by a rate adjustment reflecting personal tax effects). As the cost of debt is reflected in corporate discount rates, not cash flows, the net value of a debt equivalent is adjusted through the discount rate, not cash flows. Thus, the spirit of the traditional leasing results holds here, and lease payments are discounted at an after-tax borrowing rate, albeit one using the effective marginal tax rate rather than the statutory rate.

Let C_{t+1}, the pretax operating cash flow of the firm in period t + 1, be a continuously distributed random variable with uncertainty resolved between times t and t + 1. Investors remain risk neutral, but there is now a borrowing rate r_d and a risk-free rate $r < r_d$ applied to equity cash flows. The lower rate on equity cash flows can be motivated by differential personal taxes applied to debt and equity income, as in DeAngelo and Masulis. If τ_d and τ_e are the personal tax rates applying to debt and equity respectively, then in equilibrium $(1 - \tau_d)r_d = (1 - \tau_e)r$, and a personal tax advantage to equity ($\tau_e < \tau_d$) implies $r < r_d$. For simplicity, assume no ability to carry tax losses forward or back. With only ordinary debt available, optimum firm value is determined by

$$V_t^* = \max_{D_t \geq 0} \; D_t + E_t[C_{t+1} - (1 + r_d)D_t - \tau \cdot \mathrm{Max}(0, C_{t+1} - r_dD_t) + V_{t+1}^*]/(1 + r), \tag{7}$$

where E_t represents the expectation taken at time t. The first-order condition is $r - r_d + \tau r_d \cdot \text{Prob}(C_{t+1} > r_d D_t^*) = 0$ for optimal debt level D_t^*.

In the presence of a debt equivalent with principal payments P_s, tax deductible interest payments I_s, and other tax deductions δ_s, optimum firm value is determined by

$$\begin{aligned} V_t^{**} = \underset{D_t \geq 0}{\text{Max}} \; & D_t + E_t[C_{t+1} - (1 + r_d)D_t - (P_{t+1} + I_{t+1}) \\ & - \tau \cdot \text{Max}(0, C_{t+1} - r_d D_t - I_{t+1} - \delta_{t+1}) + V_{t+1}^{**}]/(1 + r). \end{aligned} \quad (8)$$

The first-order condition is $r - r_d + \tau r_d \,\text{Prob}(C_{t+1} > r_d D_t^{**} + I_{t+1} + \delta_{t+1}) = 0$ for the optimal debt level D_t^{**}. Thus, $r_d D_t^{**} + I_{t+1} + \delta_{t+1} = r_d D_t^*$, and tax deductions from the debt equivalent displace those of ordinary debt. Subtracting (7) from (8), and substituting recursively,

$$\begin{aligned} NV_t &= V_t^{**} - V_t^* \\ &= [-P_{t+1} - (r/r_d)I_{t+1} + (1 - r/r_d)\delta_{t+1} + (V_{t+1}^{**} - V_{t+1}^*)]/(1 + r), \\ &= \textstyle\sum_{s>t} [-P_s - (r/r_d)I_s + (1 - r/r_d)\delta_{t+1}]/(1 + r)^{s-t} \\ &= \textstyle\sum_{s>t} [-P_s - (1 - \tau^*)I_s + \tau^*\delta_s]/[1 + (1 - \tau^*)r_d]^{s-t}, \end{aligned} \quad (9)$$

where τ^* is the effective marginal tax rate (the probability of paying positive tax in a given period times the corporate tax rate τ). From first-order conditions, $\tau^* = (r_d - r)/r_d$. Here, the net value of a debt equivalent can be evaluated by discounting the after-effective-tax debt payments and tax savings at the after-effective-tax borrowing rate. For the case of a lease, after-effective-tax lease payments and foregone tax savings from lost depreciation can be discounted at the after-effective-tax borrowing rate. Two remarks are worth re-emphasizing at this point. First, it is the effective tax rate, which is generally less than the statutory rate, which must be used to adjust both the cash flows and the discount rate. Second, this result, which is similar to that of the standard lease valuation approach, arises because personal taxes do not affect the cash flows of the firm, but rather the discount rate the firm faces. This can be seen by noting that $(1 - \tau^*) = r/r_d = (1 -$

$\tau_d)/(1 - \tau_e)$, as changing either personal tax rate affects the ratio of the discount rates r and r_d.

A special case of (9) is given by the case of the Miller equilibrium. If tax rates are such that $(1 - \tau_d) = (1 - \tau_e)(1 - \tau)$, then there is no net advantage to any ordinary debt financing. Of course, a relative advantage for a leasing arrangement or below-market rate debt financing may still exist, depending upon the particulars of the contract. Since the after-personal tax returns to investors from debt and equity are equated, $(1 - \tau_d)r_d = (1 - \tau_e)r$, implying that $r = (1 - \tau)r_d$. As long as the debt level is low enough so that tax shields are fully used, thus $\mathrm{Prob}(C_{t+1} > r_d D_t + I_{t+1} + \delta_{t+1}) = 1$, the condition $r - r_d + \tau r_d \cdot \mathrm{Prob}(C_{t+1} > r_d D_t + I_{t+1} + \delta_{t+1}) = 0$ holds. The effective marginal tax rate τ^* and statutory rate τ are equated, and (9) can be interpreted as discounting after-tax cash flows at the after-tax borrowing rate, coinciding with the result in the standard leasing literature.

5. The case of tangibility constraints and leases

This section assumes that debt usage by the firm is limited by an asset tangibility constraint. If outside investors are unable to value future cash flows of the firm (for example, if there is an information asymmetry or monitoring is expensive), they may be unwilling to extend credit to the firm without sufficient collateral. Thus, the firm's level of tangible assets may create a binding constraint upon the level of debt the firm is able to achieve.[10] Leasing an asset may be a way for the firm to get around this constraint. Of course, an asset that can be leased may very well be an asset that a lender would be willing to accept as collateral for a loan, in which case the firm would be able to extend its tangible asset base, and therefore its (non-leasing) debt capacity, by purchasing rather than leasing. This section therefore considers two cases. In the first case, the firm cannot borrow against the value of the asset in question if it purchases; thus leasing effectively allows the firm to carry a higher debt load. In the second case, the firm can borrow against the value of the asset in question; either leasing or purchasing the asset increases

[10] Because of returns to scale of monitoring and transaction costs, it is likely that this constraint binds more often for smaller firms without effective access to capital markets.

the effective debt capacity of the firm. These two cases lead to different valuations for the leasing arrangement.

Suppose that the potentially leased asset is purchased at time t. Let C_s be the operating cash flow and D_s* be the (binding) debt capacity of the firm at time s. The optimal value of the firm at time t is given by

$$\begin{aligned} V_t^* &= [(1-\tau)C_{t+1} + \tau r D_t^* + V_{t+1}^*]/(1+r) \\ &= \textstyle\sum_{s>t} [(1-\tau)C_s + \tau r D_{s-1}^*]/(1+r)^{s-t}. \end{aligned} \tag{10}$$

Now, suppose the firm had instead entered into a leasing arrangement at time t, avoiding the cost Π_t of purchasing, having foregone depreciation $\delta_s \leq 0$, and agreeing to tax-deductible lease payments $I_s \geq 0$ at times s > t.

In the first case considered, the firm cannot borrow against the asset if it is owned by the firm. The firm's debt capacity is therefore the same whether the asset is owned or not. The value of the firm (excepting the leasing claim) if the asset is leased is

$$\begin{aligned} V_t^{**} &= \Pi_t + [(1-\tau)C_{t+1} - I_{t+1} + \tau(rD_t^* + I_{t+1} + \delta_{t+1}) + V_{t+1}^{**}]/(1+r) \\ &= \Pi_t + \textstyle\sum_{s>t} [(1-\tau)C_s - (1-\tau)I_s + \tau r D_{s-1}^* + \tau\delta_s)]/(1+r)^{s-t}. \end{aligned} \tag{11}$$

Subtracting (10) from (11), the net value to the firm of the leasing arrangement is

$$\begin{aligned} NV_t &= V_t^{**} - V_t^* \\ &= \Pi_t + \textstyle\sum_{s>t} [-(1-\tau)I_s + \tau\delta_s)]/(1+r)^{s-t}. \end{aligned} \tag{12}$$

The net value is calculated by discounting after-tax lease payments and foregone tax savings from depreciation, using the pre-tax borrowing rate.

In the second case considered, the firm can borrow against the asset if it is owned by the firm. The firm's debt capacity, if it leases rather than purchases the asset, is diminished by the value of the asset at each point in time. The value of the firm, excepting the leasing claim, under the leasing arrangement would be (details are in the Appendix)

$$V_t^{**} = (1 - \tau)\Pi_t + \sum_{s>t} [(1 - \tau)C_s - (1 - \tau)I_s + \tau r D_{s-1}^*]/(1 + r)^{s-t}. \tag{13}$$

Subtracting (10) from (13), the lease has a net value to the firm of

$$\begin{aligned} NV_t &= V_t^{**} - V_t^* \\ &= (1 - \tau)\Pi_t - \sum_{s>t} (1 - \tau)I_s/(1 + r)^{s-t}. \end{aligned} \tag{14}$$

Here, there are four terms affecting the value of the lease: avoiding the upfront purchase price, making tax-deductible lease payments, lost tax shields from foregone depreciation, and lost tax shields from foregone borrowing. Since the value of the asset at any time equals the purchase price less the foregone depreciation, adding the foregone tax shields from depreciation and borrowing equals the tax rate times the purchase price. Thus, in this case the net value of the lease can be simply calculated by discounting after-tax lease payments using the pre-tax borrowing rate, and comparing with this "tax-adjusted" purchase price. The results of this section are summarized in the following proposition.

Proposition 2. *Suppose a firm is constrained in its borrowing by a binding asset tangibility constraint. The net value of entering into a leasing arrangement is given by (12) when the leased asset, if owned outright, cannot be used as collateral. The net value is given by (14) when the leased asset, if owned outright, can be used as collateral.*
Proof: See Appendix.

6. Conclusion

This paper takes a fresh look at valuing corporate debt or debt equivalents, with particular emphasis on valuing leases, recognizing that the acceptance of any debt or debt equivalent crowds other debt out of the corporate capital structure. The traditional literature on lease valuation is based upon an assumption that debt generates a corporate tax benefit; calculating the marginal value of any particular debt contract to the firm requires recognition of tax benefits associated with the lost opportunity to issue an alternative debt contract. However, in the context of an optimal capital structure, this approach raises the important question of exactly what offsets the tax benefit of debt at the margin. For example, in the important case where agency or other financial distress costs associated with debt lower expected operational cash flows, then the standard approach fails to recognize all the cash flows associated with debt (only the benefits, and not the costs). This paper shows how to value a debt or debt equivalent, in particular a leasing arrangement, for a firm at an optimal capital structure, recognizing both the cash flows from tax benefits and agency effects of debt. Valuing a leasing arrangement requires discounting after-tax lease payments and tax savings from depreciation, less a term based on time-weighted lease payments, using the pre-tax borrowing rate.[11] The calculation turns out to be relatively easy to implement.

In the case when asset tangibility constrains the firm's borrowing ability, the after-tax lease payments and tax savings from depreciation should also be discounted using the pre-tax borrowing rate, possibly with an adjustment term. As in the case of agency costs, the pretax borrowing rate should be used to discount cash flows. When the optimal capital structure is driven by an adverse cash flow effect offsetting the tax benefit of debt at the margin, the appropriate discount rate to use is the pretax borrowing rate. (An asset tangibility constraint is equivalent to treating debt usage beyond the constraint point as generating an infinitely negative cash flow.)

Only when the optimal capital structure is driven by non-cash flow effects offsetting the tax benefit of debt, such as the personal tax effects of DeAngelo and Masulis, or the Miller equilibrium, does the appropriate leasing valuation methodology discounts after-effective-tax leasing payments at the after-effective-tax borrowing rate (similar to standard results on leasing valuation, but possibly requiring use of the effective tax rate). Precise implementation in this case will generally require estimating the effective tax rate, as in the work of Shevlin, Graham, and Graham et al. Only in the extreme case of the Miller equilibrium is it appropriate to use the statutory tax rate in the

[11] Thus, the calculation may be thought of as roughly analogous to the adjusted net present value methodology of Myers, with a term explicitly calculating the agency cost generated over time.

valuation calculation. At the other extreme, the Berens and Cuny case of the fully shielded firm can be interpreted as an effective tax rate of zero. Thus, the appropriate methodology for lease valuation depends critically upon what offset to corporate tax shields drives the optimal corporate capital structure.

Appendix

Proof of Proposition 1. Subtracting (1) from (3) and substituting recursively yields equation (4)

$$
\begin{aligned}
NV_t &= V_t^{**} - V_t^{*} \\
&= [-P_{t+1} - (1 - \tau)I_{t+1} + \tau\delta_{t+1} - \tau r Q_t + (V_{t+1}^{**} - V_{t+1}^{*})]/(1 + r), \\
&= \Sigma_{s>t}\, [-P_s - (1 - \tau)I_s + \tau\delta_s - \tau r Q_{s-1}]/(1 + r)^{s-t} \\
&= \Sigma_{s>t}\, [-P_s - (1 - \tau)I_s + \tau\delta_s]/(1 + r)^{s-t} \\
&\qquad - \tau r/(1 + r) \cdot \Sigma_{u \geq t}\, Q_u/(1 + r)^{u-t}. \\
&= \Sigma_{s>t}\, [-P_s - (1 - \tau)I_s + \tau\delta_s]/(1 + r)^{s-t} \\
&\qquad - \tau r/(1 + r) \cdot \Sigma_{u \geq t} \left(\Sigma_{s>u}\, (P_s + I_s)/(1 + r)^{s-u} \right)/(1 + r)^{u-t} \\
&= \Sigma_{s>t}\, [-P_s - (1 - \tau)I_s + \tau\delta_s]/(1 + r)^{s-t} \\
&\qquad - \tau r/(1 + r) \cdot \Sigma_{s>u \geq t}\, (P_s + I_s)/(1 + r)^{s-t} \\
&= \Sigma_{s>t}\, [-P_s - (1 - \tau)I_s + \tau\delta_s]/(1 + r)^{s-t} \\
&\qquad - \tau r/(1 + r) \cdot \Sigma_{s>t}\, (s - t)(P_s + I_s)/(1 + r)^{s-t}. \quad \blacklozenge
\end{aligned}
$$

Proof of Corollary A. Suppose the firm enters into a ordinary debt contract D^0 whose present value at time s is D^0_s, implying interest payments $I_s = rD^0_{s-1}$, principal payments $P_s = D^0_{s-1} - D^0_s$, and no additional tax deductions $\delta_s = 0$. Note that a repayment schedule implies that $D^0_s/(1 + r)^s \rightarrow 0$ as $s \rightarrow \infty$; this implies convergence of the telescoping series encountered.

Substituting I_s and P_s into (2) and summing the telescoping series shows that $Q_t = D^0_t$. Substituting P_s, I_s, δ_s and Q_s into (4), collecting terms, and summing the telescoping series, $V_t^{**} - V_t^{*} = -D^0_t = -Q_t$. Thus, at all times, the impact of the debt contract on the firm value is exactly the value of the debt D^0. ♦

Proof of Corollary B. This follows from equation (4), letting I_s be the lease payments and P_s be zero. ♦

Extensions of Section 3.

Agency costs affect future cash flows. Here, the case where debt can generate agency or other financial distress cost effects on both concurrent and future cash flows is considered. Suppose that, due to agency effects, current debt (and debt equivalents) can negatively affect both next period's operating cash flow and the continuation value of the firm through cash flows beyond next period. We assume the magnitude of the impact is non-decreasing in the debt, thus $dV_{t+1}/dD_t \leq 0$ and $d^2V_{t+1}/dD_t^2 \leq 0$. Optimization (1) is unchanged. Optimization (3) becomes

$$V_t^{**} = \max_{D_t \geq 0} \; [(1 - \tau)C_{t+1}[D_t + Q_t] + \tau(rD_t + I_{t+1} + \delta_{t+1}) - (P_{t+1} + I_{t+1}) + V_{t+1}^{**}[D_t + Q_t]]/(1 + r). \qquad (3')$$

The first-order condition of (1) is

$$(1 - \tau)\cdot dC_{t+1}/dD_t[D_t^*] + dV^*_{t+1}/dD_t[D_t^*] + \tau r = 0,$$

for optimal debt level D_t^*. The first-order condition of (3') is

$$(1 - \tau)\cdot dC_{t+1}/dD_t[D_t^{**} + Q_t] + dV^{**}_{t+1}/dD_t[D_t^{**} + Q_t] + \tau r = 0,$$

for optimal debt level D_t^{**}. We show that (4) holds by backward induction. Suppose (4) holds at times after t; we will show it holds at t. Since P_s, I_s, δ_s (and therefore Q_s) are already committed to, it follows from (4) that $dNV_{t+1}/dD_t = d(V_{t+1}^{**} - V_{t+1}^*)/dD_t = 0$, or $dV_{t+1}^{**}/dD_t = dV_{t+1}^*/dD_t$. The first-order condition of (3') can therefore be written

$$(1 - \tau)\cdot dC_{t+1}/dD_t[D_t^{**} + Q_t] + dV^*_{t+1}/dD_t[D_t^{**} + Q_t] + \tau r = 0.$$

Since $(1 - \tau)\cdot C_{t+1} + V^*_{t+1}$ is monotone in D_t, we have $D_t^{**} + Q_t = D_t^*$. Substituting into (3'), and subtracting (1) from (3'),

$$
\begin{aligned}
NV_t &= [\tau r(D_t^{**} - D_t^*) - P_{t+1} - (1 - \tau)I_{t+1} + \tau\delta_{t+1} + (V_{t+1}^{**} - V_{t+1}^*)]/(1 + r) \\
&= [-P_{t+1} - (1 - \tau)I_{t+1} + \tau\delta_{t+1} - \tau r Q_t + NV_{t+1}]/(1 + r) \\
&= [-P_{t+1} - (1 - \tau)I_{t+1} + \tau\delta_{t+1} - \tau r Q_t]/(1 + r) \\
&\qquad + \textstyle\sum_{s > t+1} [-P_s - (1 - \tau)I_s + \tau\delta_s - \tau r Q_{s-1}]/(1 + r)^{s-t} \\
&= \textstyle\sum_{s > t} [-P_s - (1 - \tau)I_s + \tau\delta_s - \tau r Q_{s-1}]/(1 + r)^{s-t} \\
&= \textstyle\sum_{s > t} [-P_s - (1 - \tau)I_s + \tau\delta_s]/(1 + r)^{s-t} \\
&\qquad\qquad - \tau r/(1 + r) \cdot \textstyle\sum_{s > t} (s - t)(P_s + I_s)/(1 + r)^{s-t}.
\end{aligned}
$$

Therefore, equation (4) and Proposition 1 hold at time t. ♦

Risky debt. Here, debt is allowed to be risky. Suppose that the corporate technology is such that, each period t there is a probability π that the firm continues, generating (conditional) cash flow C_{t+1} and continuation value V_{t+1}, and probability $1 - \pi$ of no further cash flows (annihilation). Let the promised interest rate be p. It follows that $1 + r = \pi(1 + p)$. The analogue to equation (1) is

$$
V_t^* = \max_{D_t \geq 0} \; \pi\big((1 - \tau)C_{t+1}[D_t] + \tau p D_t + V_{t+1}^* \big)/(1 + r),
$$

with first-order condition $(1 - \tau)C_{t+1}'[D_t^*] + \tau p = 0$. The analogue to (3) is

$$
\begin{aligned}
V_t^{**} = \max_{D_t \geq 0} \; \pi\big(&(1 - \tau)C_{t+1}[D_t + Q_t] + \tau(pD_t + I_{t+1} + \delta_{t+1}) \\
&- (P_{t+1} + I_{t+1}) + V_{t+1}^{**} \big)/(1 + r),
\end{aligned}
$$

where $Q_t = \sum_{s > t} (P_s + I_s)\pi^{s-t}/(1 + r)^{s-t} = \sum_{s > t} (P_s + I_s)/(1 + p)^{s-t}$. The first-order condition is $(1 - \tau)C_{t+1}'[D_t^{**} + Q_t] + \tau p = 0$. The analogue to (4) is

$$
\begin{aligned}
NV_t &= V_t^{**} - V_t^* \\
&= \textstyle\sum_{s > t} \big(-P_s - (1 - \tau)I_s + \tau\delta_s \big)/(1 + p)^{s-t} \\
&\qquad\qquad - \tau p/(1 + p) \cdot \textstyle\sum_{s > t} (s - t)(P_s + I_s)/(1 + p)^{s-t}.
\end{aligned}
$$

Thus, Proposition 1 still holds, since in this context, p (the promised interest rate) is the borrowing rate instead of r. ♦

Proof of Proposition 2. Equation (12) is derived in the text. For equation (13), assuming that the tax depreciation coincides with the economic wear of the asset, the debt capacity associated with ownership of the asset at time s equals the purchase price less any accumulated depreciation, $\Pi_t + \sum_{u \leq s} \delta_u$. Therefore, if the firm leases the asset, its debt capacity at time s equals $D_s^{**} = D_s^{*} - \Pi_t - \sum_{u \leq s} \delta_u$. The value of the firm, excepting the leasing claim, under this arrangement is then

$$
\begin{aligned}
V_t^{**} &= \Pi_t + \big((1 - \tau)C_{t+1} - I_{t+1} + \tau(rD_t^{**} + I_{t+1} + \delta_{t+1}) + V_{t+1}^{**}\big)/(1 + r) \\
&= \Pi_t + \textstyle\sum_{s > t} \big((1 - \tau)C_s - (1 - \tau)I_s + \tau rD_{s-1}^{**} + \tau\delta_s\big)/(1 + r)^{s - t} \\
&= \Pi_t + \textstyle\sum_{s > t} \big((1 - \tau)C_s - (1 - \tau)I_s + \tau rD_{s-1}^{*} + \tau\delta_s\big)/(1 + r)^{s - t} \\
&\qquad - \tau r \textstyle\sum_{s > t} (\Pi_t + \sum_{u < s} \delta_u)/(1 + r)^{s - t} \\
&= \Pi_t + \textstyle\sum_{s > t} \big((1 - \tau)C_s - (1 - \tau)I_s + \tau rD_{s-1}^{*} + \tau\delta_s\big)/(1 + r)^{s - t} \\
&\qquad - \tau\Pi_t - \tau\textstyle\sum_{s > t} \delta_s/(1 + r)^{s - t} \\
&= (1 - \tau)\Pi_t + \textstyle\sum_{s > t} \big((1 - \tau)C_s - (1 - \tau)I_s + \tau rD_{s-1}^{*}\big)/(1 + r)^{s - t},
\end{aligned}
$$

which is equation (13). Equation (14) is derived in the text. ♦

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